Therefore, we computed the masses of H, D, and C^{12} from these doublets combined with a value 1.552±0.002 mMU for H_2-D; the results are collected in column Q, Table III. For comparison we also included values computed from Nier's and Roberts' energy cycles, using however the reaction energy value for $C^{12}H_2-O^{16}$ (columns Nr and Rr). It is seen, that the values of Roberts' cycle are in agreement with the nuclear reaction data, but in Nier's cycles there remains an unsolved discrepancy.

We thank Professor C. J. Bakker for his interest in this work.

* Financially supported by the foundation F.O.M.

[1] A. O. Nier, Phys. Rev. **81**, 624 (1951).
[2] T. R. Roberts, Phys. Rev. **81**, 624 (1951).
[3] A. O. Nier and T. R. Roberts, Phys. Rev. **81**, 507 (1951).
[4] J. Mattauch and A. Flammersfeld, Isotopenbericht (1949).
[5] H. Ewald, Z. Naturforsch. **5a**, 1 (1950).
[6] A. H. Wapstra, Physica **16**, 611 (1950).
[7] K. Bol, Phys. Rev. **80**, 298 (1950).
[8] E. Bergstrand, Arkiv. Fys. **2**, 119 (1950).
[9] Sommer, Thomas, and Hipple, Phys. Rev. **80**, 487 (1950).
[10] D. Norman Craig and J. J. Hoffman, Phys. Rev. **80**, 487 (1950).
[11] Hornyak, Lauritsen, Morrison, and Fowler, Revs. Modern Phys. **22**, 291 (1950).
[12] M.I.T. group; private communications of P. M. Endt and D. M. Van Patter.

Note on the General Theory of Scattering

M. L. GOLDBERGER
Institute for Nuclear Studies, University of Chicago, Chicago, Illinois
(Received April 9, 1951)

THE usual presentation of scattering theory proceeds from a discussion of the integral equations[1]

$$\Psi_a^{(\pm)}=\Phi_a+\frac{1}{E_a\pm i\epsilon-H_0}H_1\Psi_a^{(\pm)}. \tag{1}$$

The transition probability per unit time from a state characterized by Φ_a to an initially unoccupied state Φ_b may be expressed as

$$\begin{aligned} w_{ba}&=(2\pi/\hbar)\,|\,\mathbf{R}_{ba}\,|^2\delta(E_b-E_a)\\ \mathbf{R}_{ba}&=-(\Phi_b,\,H_1\Psi_a^{(+)})=-(\Psi_b^{(-)},\,H_1\Phi_a). \end{aligned} \tag{2}$$

The matrix $\mathbf{R}_{ba}$ is defined only on the energy shell $E_a=E_b=E$. (Bold-face quantities hereafter always refer to the energy shell, E.) The integral equation (1) may be replaced by a pair of integral equations: Introduce a "standing wave" state vector $\Psi_a^{(1)}$ which, except for a normalizing factor, is $(\Psi_a^{(+)}+\Psi_a^{(-)})/2$ by the equation

$$\Psi_a^{(1)}=\Phi_a+P\left(\frac{1}{E_a-H_0}\right)H_1\Psi_a^{(1)}. \tag{3}$$

It is then easy to show that (1) is satisfied if

$$\Psi_a^{(+)}=\Psi_a^{(1)}+i\pi\,\Sigma_c\,\Psi_c^{(1)}\delta(E_a-E_c)\mathbf{R}_{ca} \tag{4}$$

from which, using (2), we find the Heitler integral equation[2]

$$\mathbf{R}_{ba}=\mathbf{G}_{ba}+i\pi\,\Sigma_c\,\mathbf{G}_{bc}\delta(E-E_c)\mathbf{R}_{ca}, \tag{5}$$

with

$$\mathbf{G}_{ba}=-(\Phi_b,\,H_1\Psi_a^{(1)})=-(\Psi_b^{(1)},\,H_1\Phi_a)=\mathbf{G}_{ab}{}^*. \tag{6}$$

From (3) we obtain an integral equation for G (*not* on the energy shell):[3]

$$\begin{aligned} G_{ba}&=B_{ba}-P\,\Sigma_c\frac{B_{bc}G_{ca}}{E_a-E_c},\\ B_{ba}&=-(\Phi_b,\,H_1\Phi_a). \end{aligned} \tag{7}$$

B_{ba} is the Born approximation matrix element of the elementary scattering act. It should be carefully noted that G in contrast to $\mathbf{G}$ is *not* hermitian. This may in fact be utilized to derive an expression for the energy dependence of G near the energy shell.

A formulation of scattering theory in which (5) and (7) are regarded as fundamental rather than (1) is attractive for several reasons: (1) Only the existence of the wave functions Φ_a of the separated system, needed for finding B_{ba}, is presumed. (2) Approximation procedures based on approximate solution of (7) and exact solution of (5) have proved very useful in specific applications mentioned below and in all cases have the property of preserving the unitarity of the collision matrix. (3) The whole formalism may be derived from variational principles which are very useful in applications.

A variational basis for (5) and (7) will now be presented. Recalling that for a large class of problems the collision matrix

$$S_{ba}=\delta_{ba}+2\pi i\delta(E_b-E_a)\mathbf{R}_{ba} \tag{8}$$

is both symmetric and unitary, that consequently $\mathbf{R}$ is symmetric, and finally that $\mathbf{G}$ is real and symmetric,[4] it is easy to show[5] that the matrix I,

$$I=\mathbf{RG}+\mathbf{GR}+i\pi\mathbf{RGR}-\mathbf{RR}, \tag{9}$$

with matrix multiplication defined by

$$(\mathbf{RG})_{ba}=\Sigma_c\,\mathbf{R}_{bc}\delta(E-E_c)\mathbf{G}_{ca},\ \text{etc.},$$

is stationary under arbitrary variations about the correct $\mathbf{R}$. Conversely, the requirement of vanishing variation leads to (5). An alternate normalization independent expression of the same stationary principle is

$$I_{ba}'=(\mathbf{RG})_{ba}(\mathbf{GR})_{ba}[(\mathbf{RR})_{ba}-i\pi(\mathbf{RGR})_{ba}]^{-1}. \tag{10}$$

The stationary value obtained (the same for (9) and (10)) is $(\mathbf{R}-\mathbf{G})/i\pi$. These expressions have proved very useful for approximate determinations of $\mathbf{R}$. They have also been used to investigate the nature of the errors involved using the Heitler prescription of replacing $\mathbf{G}$ by $\mathbf{B}$.[6] An analogous pair of relations may be given for $\mathbf{G}$. Consider the matrix J_{ba} with $E_a=E_b=E$ defined by

$$\begin{aligned} J_{ba}=P\,\Sigma_c\frac{G_{bc}{}^\dagger G_{ca}}{E-E_c}+P\,\Sigma_{c,d}\frac{G_{bc}{}^\dagger B_{cd}G_{da}}{(E-E_c)(E-E_d)}\\ -P\,\Sigma_c\frac{G_{bc}{}^\dagger B_{ca}+B_{bc}G_{ca}}{E-E_c} \end{aligned} \tag{11}$$

J_{ba} is stationary under arbitrary variations about the correct G and $G^\dagger$, the demand of vanishing variation yields (7) and its adjoint. A normalization independent form of (11) is

$$J_{ba}'=-(G^\dagger B)_{ba}(BG)_{ba}[(G^\dagger G)_{ba}+(G^\dagger BG)_{ba}]^{-1}, \tag{12}$$

where matrix multiplication is defined by

$$(G^\dagger G)_{ba}=P\,\Sigma_c\,G_{bc}{}^\dagger G_{ca}/(E-E_c),\ \text{etc.}$$

The stationary value of J and J' is $\mathbf{G}-\mathbf{B}$.

A detailed discussion of the above material together with the results of applications to nucleon-nucleon and meson-nucleon scattering problems, and to the nonrelativistic Compton effect will be presented shortly.

[1] B. Lippmann and J. Schwinger, Phys. Rev. **79**, 469 (1950). We follow the notation of these authors except $\mathbf{T}\to-\mathbf{R}$, $\mathbf{K}\to-\mathbf{G}$.
[2] See, for example, W. Pauli, *Meson Theory of Nuclear Forces* (Interscience Publishers, Inc., New York).
[3] Equation (7) is usually given only in iterated form, reference 2; this is neither desirable nor necessary.
[4] J. A. Wheeler, Phys. Rev. **52**, 1107 (1937); W. Heisenberg, Z. Physik **120**, 513, 673 (1943); E. P. Wigner and L. Eisenbud, Phys. Rev. **72**, 29 (1947). In cases for which S is not symmetric in the elementary sense, a somewhat more complicated formulation can be made. This will be discussed in a later publication.
[5] See also, S. T. Ma and C. F. Hsueh, Phys. Rev. **67**, 303 (1945). I am indebted to Dr. Ma for a discussion of this work and the work of the present paper.
[6] W. Heitler, *Quantum Theory of Radiation* (Oxford University Press, London, 1944), second edition.

A Comparison of Theories of Secondary Emission

JAMES J. BROPHY
Physics Department Illinois Institute of Technology, Chicago, Illinois
(Received April 13, 1951)

THE theory of secondary emission using a quantum-mechanical attack has achieved reasonable success in predicting the variation of the secondary emission ratio with primary energy for several materials.[1] Recently an admittedly approximate theory using the free-electron approximation has appeared.[2] It is